

AN IDENTIFICATION FOR EISENSTEIN POLYNOMIALS OVER A p -ADIC FIELD

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In this note, we give a criteria whether given two Eisenstein polynomials over a p -adic field define the same extension (Proposition 1.6). In particular, we completely identify Eisenstein polynomials of degree p (Theorem 1.16). This note is an English translation of a part of [12].

1. EISENSTEIN POLYNOMIALS AND RAMIFICATION THEORY

In Section 1.1, we consider general Eisenstein polynomials. In Section 1.2, we precisely investigate Eisenstein polynomials of degree p over \mathbb{Q}_p .

1.1. General Eisenstein polynomials. In this subsection, we assume K is a finite extension of \mathbb{Q}_p ¹. We fix an algebraic closure \overline{K} of K and we assume throughout that all algebraic extensions of K under discussion are contained in \overline{K} . We denote by \mathcal{O}_K the valuation ring of K and by v_K the valuation on \overline{K} such that $v_K(K^\times) = \mathbb{Z}$. Let L be a finite separable extension of K . We denote by \mathcal{O}_L the integral closure of \mathcal{O}_K in L . There exists an element $\alpha \in \mathcal{O}_L$ such that $\mathcal{O}_L = \mathcal{O}_K[\alpha]$ (the existence of such an element is proved in [10], Chap. III, Sect. 6, Prop. 12). Put $H = \text{Hom}_K(L, \overline{K})$. The order function $\mathbf{i}_{L/K}$ is defined on H by

$$\mathbf{i}_{L/K}(\sigma) = v_K(\sigma(\alpha) - \alpha)$$

for any $\sigma \in H$. This function is independent of the choice of α . The i th *lower numbering ramification set* $H_{(i)}$ of H are defined for a real number $i \geq 0$ by

$$H_{(i)} = \{\sigma \in H \mid \mathbf{i}_{L/K}(\sigma) \geq i\}.$$

The transition function $\tilde{\varphi}_{L/K} : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ of L/K is defined by

$$\tilde{\varphi}_{L/K}(i) = \int_0^i \#H_{(t)} dt$$

for any real number $i \geq 0$, where $\#H_{(t)}$ is the cardinality of $H_{(t)}$. Its inverse function is denoted by $\tilde{\psi}_{L/K}$. Then the u th *upper numbering ramification set* $H^{(u)}$ of H are defined for a real number $u \geq 0$ by

$$H^{(u)} = H_{(i_0)}, \quad i_0 := \tilde{\psi}_{L/K}(u)$$

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¹ The results in Subsection 1.1 hold for any complete discrete valuation field K with perfect residue field of characteristic p and any finite separable extension L/K .

A *ramification break* is a real number i (resp. u) such that $H_{(i)} \neq H_{(i+\varepsilon)}$ (resp. $H^{(u)} \neq H^{(u+\varepsilon)}$) for any $\varepsilon > 0$. Denote the largest lower (resp. upper) numbering ramification break by

$$i_{L/K} = \inf\{i \in \mathbb{R} \mid H_{(i)} = 1\}, \quad u_{L/K} = \inf\{u \in \mathbb{R} \mid H^{(u)} = 1\}.$$

Remark 1.1. If L/K is a Galois extension, then our filtration $H^{(u)}$ coincides with the filtration shifted by one defined in [10], Chapter IV.

Proposition 1.2 ([2], Prop. A.6.1). *Let $\mathfrak{D}_{L/K}$ be the different of L/K . Then we have*

$$u_{L/K} = i_{L/K} + v_K(\mathfrak{D}_{L/K}).$$

In particular, if L/K has only one ramification break, then the above shows

$$u_{L/K} = e \cdot i_{L/K} = e/(e-1) \cdot v_K(\mathfrak{D}_{L/K}),$$

where e is the ramification index of L/K .

Remark 1.3. The Galois case is proved by [4], Proposition 1.3.

Lemma 1.4 ([2], Prop. A.6.2). *Let L be a finite separable extension of K . Put $H = \text{Hom}(L, \overline{K})$. Choose an element $\alpha \in \mathcal{O}_L$ such that $\mathcal{O}_L = \mathcal{O}_K[\alpha]$. Let f be the minimal polynomial of α over K and β an element of Ω . Put $i = \sup_{\sigma \in H} v_K(\sigma(\alpha) - \beta)$ and $u = v_K(f(\beta))$. Then we have*

$$u = \tilde{\varphi}_{L/K}(i), \quad \tilde{\psi}_{L/K}(u) = i.$$

Remark 1.5. The numbering of the ramification filtration in [2] is different from ours. We adopt the numbering in [4] since it is suitable for Proposition 1.6, which is repeatedly used in this paper.

Let E_K^e be the set of all Eisenstein polynomials of degree e over K . For two polynomials $f, g \in E_K^e$, we put

$$v_K(f, g) = \min_{0 \leq i \leq e-1} \{v_K(a_i - b_i) + \frac{i}{e}\}.$$

Then we have $v_K(f, g) = v_K(f(\pi_g))$ for any root π_g of g , and $v_K(\cdot, \cdot)$ defines an ultrametric on E_K^e (cf. [7], [9]). For each $f \in E_K^e$, we put $L_f = K[X]/(f)$ and $u_f = u_{L_f/K}$. For any $f, g \in E_K^e$, we define an equivalence $f \sim g$ on E_K^e by the existence of a K -isomorphism $L_f \cong L_g$.

Proposition 1.6. *Let $f, g \in E_K^e$. If $v_K(f, g) > u_f$, then we have $f \sim g$.*

Proof. Take a root π_g of g and choose a root π_f of f such that $v_K(\pi_g - \pi_f)$ is the maximum. Put $u_f = u_{L_f/K}$, $i_f = i_{L_f/K}$, $\tilde{\psi}_f = \tilde{\psi}_{L_f/K}$. By assumption, we have $v_K(f, g) = v_K(f(\pi_g)) > u_f$. Note that $\mathcal{O}_{L_f} = \mathcal{O}_K[\pi_f]$. Mapping the equation by $\tilde{\psi}_f$ gives an inequality

$$v_K(\pi_g - \pi_f) = \tilde{\psi}_f(v_K(f(\pi_g))) > \tilde{\psi}_f(u_f) = i_f = \sup_{\sigma \in H, \sigma \neq 1} v_K(\sigma(\pi_f) - \pi_f)$$

by Lemma 1.4. We have $L_f \cong K(\pi_f) \subset K(\pi_g) \cong L_g$ by Krasner's lemma. Since their degrees are the same, we obtain an isomorphism $L_f \cong L_g$. \square

Remark 1.7. The case where L_f/K is a Galois extension is proved in [11], Proposition 3.1.

1.2. Degree p . In this subsection, we assume that p is odd². We assume that the base field is \mathbb{Q}_p and denote the p -adic valuation by v_p .

Proposition 1.8 ([1], Thm. 6 and 7). *Suppose that p is odd. Table 1.1 gives exactly one polynomial for each isomorphism class of totally ramified extension of \mathbb{Q}_p of degree p . In the table, we put $d_f = v_p(\mathfrak{D}_{L_f/\mathbb{Q}_p})$.*

Family	Parameter	d_f	u_f
$X^p + apX^\lambda + p$	$1 \leq a \leq p-1$ $1 \leq \lambda \leq p-1$ $(\lambda, a) \neq (p-1, p-1)$	$1 + \frac{\lambda-1}{p}$	$1 + \frac{\lambda}{p-1}$
$X^p - pX^{p-1} + (1+ap)p$	$0 \leq a \leq p-1$	$1 + \frac{p-2}{p}$	2
$X^p + (1+ap)p$	$0 \leq a \leq p-1$	$1 + \frac{p-1}{p}$	$2 + \frac{1}{p-1}$

TABLE 1.1. A complete system of representatives of $E_{\mathbb{Q}_p}^p / \sim$

Remark 1.9 ([6], Prop. 2.3.1 for details). (i) The Galois group of a polynomial f of the first type in Table 1.1 is a semi-direct product $C_p : C_{d_2}$, where $d_2 = (p-1)/\gcd((p-1)/m, g)$, $g = \gcd(p-1, \lambda)$ and m is the order of $a\lambda$ in \mathbb{F}_p^\times . Moreover, its inertia group is $C_p : C_{d_1}$, where $d_1 = (p-1)/g$. The second type in Table 1.1 is the only case that L_f/\mathbb{Q}_p is a cyclic. The Galois group and its inertia subgroup of the third type in Table 1.1 are $C_p : C_{p-1}$.

(ii) More precisely, in [1], the explicit description of the Galois closure of L_f/\mathbb{Q}_p as $\mathbb{Q}_p(\pi_f, \gamma)$ where $\gamma^{p-1} \in \mathbb{Q}_p$. An algorithm for computing the automorphism group of a finite extension L/\mathbb{Q}_p has been implemented in Magma as the inner function `AutomorphismGroup(L, \mathbb{Q}_p)`, where the output is given as a subgroup of the symmetric group S_p . Hence we can explicitly calculate the Galois group of f .

Let $f = X^p + a_{p-1}X^{p-1} + \cdots + a_1X + a_0 \in E_{\mathbb{Q}_p}^p$. We say that f is of type $\langle 0 \rangle$ if $v_p(a_i) \geq 2$ for any i . If $v_p(a_i) = 1$ for some i , then we put $\lambda := \min\{1 \leq i \leq p-1 \mid v_p(a_i) = 1\}$. In this case, we say that f is of type $\langle \lambda \rangle$. Then we see

$$d_f = \begin{cases} 1 + (\lambda-1)/p & f \text{ is of type } \langle \lambda \rangle \\ 1 + (p-1)/p & f \text{ is of type } \langle 0 \rangle. \end{cases}$$

The type of f depends only on its equivalence class since d_f does also.

Lemma 1.10 ([1], Lemma 1). *For any totally ramified extension L/K of degree p has only one ramification break.*

By Proposition 1.2 and Lemma 1.10, we have

$$u_f = \begin{cases} 1 + \lambda/(p-1) & f \text{ is of type } \langle \lambda \rangle \\ 2 + 1/(p-1) & f \text{ is of type } \langle 0 \rangle. \end{cases}$$

² We can easily check the isomorphy of quadratic extensions, so that we consider only odd primes.

Proposition 1.11. *For $f = \sum_i a_i x^i$, $g = \sum_i b_i x^i \in E_{\mathbb{Q}_p}^p$, if one of the following conditions is satisfied, then we have $f \sim g$.*

- (i) *Both f and g are of type $\langle \lambda \rangle$, $\lambda < p-1$ and $v_p(a_i - b_i) \geq 2$ ($i = 0, \lambda$).*
- (ii) *Both f and g are of type $\langle p-1 \rangle$ and $v_p(a_i - b_i) \geq 3$ ($i = 0, p-1$).*
- (iii) *Both f and g are of type $\langle 0 \rangle$ and $v_p(a_i - b_i) \geq 3$ ($i = 0, 1$).*

Proof. In each case, it is enough to show $v_p(f, g) > u_f$ by Proposition 1.6. In case (i), by assumption, we have

$$v_p(f, g) = \min_{1 \leq i \leq p-1} \left\{ v_p(a_i - b_i) + \frac{i}{p} \right\} \geq \min \left\{ 2, 1 + \frac{\lambda+1}{p} \right\} > 1 + \frac{\lambda}{p-1} = u_f.$$

Similarly, in case (ii), we have

$$v_p(f, g) = \min_{1 \leq i \leq p-1} \left\{ v_p(a_i - b_i) + \frac{i}{p} \right\} \geq 2 + \frac{1}{p} > 1 + \frac{\lambda}{p-1} = u_f.$$

Finally, in case (iii), note that $v_p(a_i - b_i) \geq 2$ for any i , so that we have

$$v_p(f, g) = \min_{1 \leq i \leq p-1} \left\{ v_p(a_i - b_i) + \frac{i}{p} \right\} \geq 2 + \frac{2}{p} > 2 + \frac{1}{p-1} = u_f,$$

where the last inequality follows from the oddness of p . \square

Corollary 1.12. *Let $f = \sum_i a_i x^i \in E_{\mathbb{Q}_p}^p$.*

- (i) *If f is of type $\langle \lambda \rangle$, then $f \sim x^p + a_\lambda x^\lambda + a_0$.*
- (ii) *If f is of type $\langle 0 \rangle$, then $f \sim x^p + a_1 x + a_0$. Furthermore, if $v_p(a_1) \neq 2$, then $f \sim x^p + a_0$.*

To consider the case where f is of type $\langle 0 \rangle$ and $v_p(a_1) = 2$, we need some device.

Lemma 1.13. *Let $f \in E_{\mathbb{Q}_p}^p$ and π be a root of f .*

- (i) *For $u \in \mathbb{Z}_p^\times$, if we put $\pi' = \pi + u\pi^2$, then we have $\mathbb{Q}_p(\pi') = \mathbb{Q}_p(\pi)$.*
- (ii) *Take the minimal polynomial $g \in E_{\mathbb{Q}_p}^p$ of π' . Then we have*

$$g(ux^2 + x) = -u^p f(x) f(-x - u^{-1}).$$

Proof. (i) is trivial. We prove (ii). Let $\pi_1, \pi_2, \dots, \pi_p$ be the conjugate elements of π over \mathbb{Q}_p . Then the conjugate elements of π' are $\pi_1 + u\pi_1^2, \pi_2 + u\pi_2^2, \dots, \pi_p + u\pi_p^2$. For all $1 \leq i \leq p$, multiply the equations

$$(ux^2 + x) - (\pi_i - u\pi_i^2) = (x - \pi_i)(ux + 1 + u\pi_i) = -u(x - \pi_i) \{(-x - u^{-1}) - \pi_i\},$$

then we have the result. \square

Proposition 1.14. *Let $f = \sum_i a_i x^i \in E_{\mathbb{Q}_p}^p$. If f is of type $\langle 0 \rangle$ and $v_p(a_1) = 2$, then we have*

$$f \sim x^p + a_0(1 - u^p a_0),$$

where we put $u = -a_1/(pa_0)$.

Proof. By Corollary 1.12 (ii), we have $f \sim f_1 := x^p + a_1 x + a_0$. Take a root π of f_1 . Let $g_1 = \sum_i b_i x^i$ be the minimal polynomial of $\pi + u\pi^2$ over \mathbb{Q}_p . By Lemma 1.13 (i), we have $f_1 \sim g_1$ and by (ii), an equality

$$g_1(ux^2 + x) = -u^p f_1(x) f_1(-x - u^{-1})$$

holds. By comparing the coefficients of x^0 and x^1 in the both-hand side, we have

$$b_1 = a_1 + upa_0 + u^{p-1}a_1^2 = u^{p-1}a_1^2, \quad b_0 = a_0 + u^{p-1}a_0a_1 - u^p a_0^2.$$

Since the type is independent of equivalence classes, g_1 is also of type $\langle 0 \rangle$. By the inequality $v_p(b_1) \geq 3$, Proposition 1.11 (iii) gives the equivalence

$$g_1 \sim x^p + b_0$$

follows. By assumption, we note that $v_p(a_0 a_1) \geq 3$, so that Proposition 1.11 (iii) gives the equivalence

$$x^p + b_0 \sim x^p + a_0(1 - u^p a_0).$$

□

The following lemma is a result in field theory:

Lemma 1.15. *Let $f = X^e + a_{e-1}X^{e-1} + \cdots + a_1X + a_0 \in E_K^e$ and π_f a root of f . Then, for any $u \in U_K$, the Eisenstein polynomial of $u\pi_f$ over K is*

$$X^e + ua_{e-1}X^{e-1} + u^2a_{e-2}X^{e-2} + \cdots + u^{e-1}a_1X + u^ea_0.$$

Proof. This is trivial. □

By the following theorem, we can identify a given polynomial as the one in Table 1.1:

Theorem 1.16. *Let $f = \sum_i a_i x^i \in E_{\mathbb{Q}_p}^p$. If $a_i \neq 0$, we put $u_i = a_i/p^{s_i}$ ($s_i = v_p(a_i)$). For $u \in \mathbb{Z}_p^\times$ and $n \in \mathbb{Z}_{\geq 1}$, we denote by $\langle u \bmod p^n \rangle$ the integer i such that $u \equiv i \pmod{p^n}$ and $1 \leq i \leq p^n - 1$. Put*

$$u = \langle u_0^{-1} \bmod p \rangle, \quad a'_0 = u^p a_0, \quad a''_0 = a_0 \{1 + (u_0^{-1} u_1)^p a_0\},$$

$$u'_\lambda = u^{p-\lambda} u_\lambda \text{ and } a'_\lambda = u'_\lambda p.$$

(i) *If f is of type $\langle \lambda \rangle$, then*

$$f \sim \begin{cases} f_1 := X^p + \langle a'_\lambda \bmod p^2 \rangle \cdot X^\lambda + p & \text{if } \lambda \neq p-1 \text{ or } u'_\lambda \not\equiv -1 \pmod{p}, \\ f_2 := X^p - pX^{p-1} + \langle a'_0 \bmod p^3 \rangle & \text{if } \lambda = p-1 \text{ and } u'_\lambda \equiv -1 \pmod{p}. \end{cases}$$

(ii) *If f is of type $\langle 0 \rangle$, then*

$$f \sim \begin{cases} f_3 := X^p + \langle a'_0 \bmod p^3 \rangle & \text{if } v_p(a_1) \neq 2, \\ f_4 := X^p + \langle a''_0 \bmod p^3 \rangle & \text{if } v_p(a_1) = 2. \end{cases}$$

Proof. (i) We assume that f is of type $\langle \lambda \rangle$. Then we have $f \sim g_1 := x^p + a_\lambda x^\lambda + a_0$ by Corollary 1.12 (i). Apply Lemma 1.15 to g_1 , we have $g_1 \sim g_2 := x^p + u^{p-\lambda} a_\lambda x^\lambda + u^p a_0$.

First, we show the case $\lambda \neq p-1$ or $u'_\lambda \not\equiv -1 \pmod{p}$. In case $\lambda \neq p-1$, we note that $a'_0 \equiv p \pmod{p^2}$, so that we have

$$v_p(g_2, f_1) \geq 2 > 1 + \frac{\lambda}{p-1} = u_f.$$

Hence we have $g_2 \sim f_1$ by Proposition 1.6, and an equivalence $f \sim f_1$ follows it. In case $\lambda = p-1$ and $u'_\lambda \not\equiv -1 \pmod{p}$, we note that ${}^{p-1}\sqrt{\lambda u'_\lambda} \notin \mathbb{F}_p$. By Proposition 1.6 and 1.17 below (as $m = 1$, $\omega = \lambda u'_\lambda$, $\pi_K = p$), we have $g_2 \sim g_3 := x^p + a'_\lambda x^\lambda + p$. By the inequality $v_p(g_3, f_1) > 2$ and Proposition 1.6, we have $g_3 \sim f_1$, so that $f \sim f_1$.

Second, we prove the case $\lambda = p-1$ and $u'_\lambda \equiv -1 \pmod{p}$. By the inequality

$$v_p(g_3, f_2) \geq 2 + (p-1)/p > 2 = u_f$$

and Proposition 1.6, we have $g_3 \sim f_2$. Hence we obtain an equivalence $f \sim f_2$.

(ii) Suppose that f is of type $\langle 0 \rangle$ and $v_p(a_1) \neq 2$. According to Corollary 1.12 (ii), we have $f \sim g_4 := X^p + a_0$. Apply Lemma 1.15 to f_0 with similar argument as in the proof of (i), then we have $g_4 \sim g_5 := X^p + a'_0$. Note that $v_p(g_5, f_3) \geq 3 > u_f$, so that Proposition 1.6 gives $g_5 \sim f_3$. Thus we deduce the desired equivalence $f \sim f_3$.

Second, we suppose that f is of type $\langle 0 \rangle$ and $v_p(a_1) = 2$. By Proposition 1.14, we have $f \sim g_6 := X^p + a''_0$. Note that $v_p(g_6, f_4) \geq 3 > u_f$, thus Proposition 1.6 shows $g_6 \sim f_4$. Therefore, we have $f \sim f_4$. \square

Proposition 1.17 ([1], Prop. 5). *Consider two polynomials*

$$f = x^p + sp^m x^{p-1} + tp \text{ and } g = x^p + sp^m x^{p-1} + t'p \quad (s, t, t' \in \mathbb{Z}_p^\times)$$

in $E_{\mathbb{Q}_p}^p$. Suppose $t \equiv t' \equiv 1 \pmod{p}$. If $v_p(t - t') = u_f - 1$ and $s \not\equiv -1 \pmod{p}$, then we have $f \sim g$.

1.3. Appendix: An algorithm computing the ramification breaks. Let K be a finite extension of \mathbb{Q}_p and f an Eisenstein polynomial over K . Put $H = \text{Hom}_K(L_f, \bar{K})$. Let H_i, H^u be the lower and upper numbering ramification sets of H in the sense of [2], Appendice. We give an algorithm for computing the breaks of H_i and H^u . If L_f/K is a Galois extension, then the ramification sets coincides with the ramification groups in the sense of [10]. Let i_1, i_2, \dots, i_m (resp. u_1, u_2, \dots, u_m) be the lower (resp. upper) numbering ramification breaks.

Algorithm 1.18. Input: K, f

Output: $\{(i_1, \#H_{i_1}), (i_2, \#H_{i_2}), \dots, (i_m, \#H_{i_m})\}$

- Let N be the Newton polygon of $f(x + \pi_f) \in L_f[x]$.
- Let $s_1 > s_2 > \dots > s_m$ be the slopes of N .
- Let $1 < x_1 < x_2 < \dots < x_m$ be the x -coordinates of the vertexes of N .
- Return $\{(-s_1 - 1, x_1), (-s_2 - 1, x_2), \dots, (-s_m - 1, x_m)\}$.

Remark 1.19. The Newton polygon can be computed by Magma as the inner function `NewtonPolygon(h(x))`. However, in fact, the valuations of the coefficients of $f(x + \pi_f) = \sum_{i=1}^e b_i x^i$ can be directly written as

$$v_{L_f}(b_i) = \min_{i \leq j \leq e} \{v_{L_f}(a_j) + v_{L_f}\left(\binom{j}{i}\right) + (j - i)\},$$

where we write $f = \sum_{i=0}^e a_i x^i$.

Algorithm 1.20. Input: $\{(i_1, \#H_{i_1}), (i_2, \#H_{i_2}), \dots, (i_m, \#H_{i_m})\}$

Output: $\{u_1, u_2, \dots, u_m\}$

- Put $u_1 = i_1$.
- If $m = 1$ then:
 - Return $\{u_1\}$.
- If $m \geq 2$ then:
 - $S \leftarrow \{u_1\}$.
 - $s \leftarrow 2$.
 - While $s \leq m$:
 - $u_s = (i_s - i_{s-1})\#H_{i_s}/\#H_{i_1} + u_{s-1}$.
 - $s \leftarrow s + 1$.
 - $S \leftarrow S \cup \{u_s\}$.
 - Return S .

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